Set theory :

A set is well defined collection or aggregate of objects having given properties and specified according to a well defined rule. The objects comprising the set are known as its elements.

Null set : A set having no element at all is called a Null or an empty set.

Subset : A set A is said to be a proper subset of B if every element of A is also an element of B and there is at least one element of B which is not an element of A and we write $A \subset B$.

if latter restriction is removed, the A is said to be a subset of B ~~and~~ $[A \subseteq B]$.

Equality of Two sets :

Two sets A and B are said to be equal, if every element of A is an element of B and if every element of B is an element of A. $[A = B]$

Universal Set :

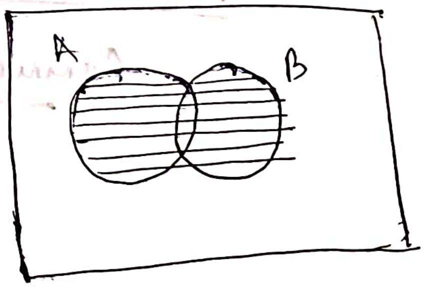
In any problem, the overall limiting set, of which all the sets under consideration are subsets, is called as universal set. $[S]$.

Union of two sets :

AUB is defined as a set of elements which belong to either A or B or Both.

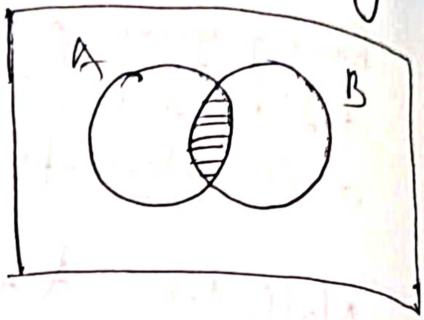
Ex. $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$

$\Rightarrow A \cup B = \{1, 2, 3, 4, 5, 6\}$



Intersection of two sets:

$A \cap B$ is defined as a set whose elements belong to both A and B.



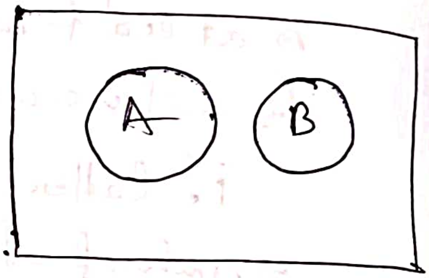
Ex: $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

Disjoint Set:

Two sets are said to be disjoint or mutually exclusive if they do not have any common point.

Ex: $A = \{1, 2\}$ $B = \{5, 6\}$

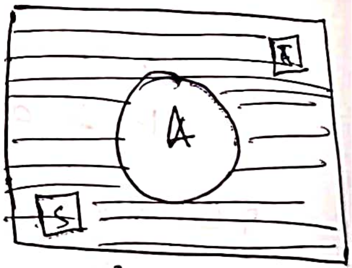


$A \cap B = \phi$.

Complement of a set:

$\bar{A}$ or A' or A^c is the set of elements which do not belong to the set A but which belong to the universal set S.

– Obviously A and A^c are disjoint set.



Difference of two sets:

$A - B$ is the set of elements which belong to A but not to B.

$A - B$ is equivalent to $A \cap \bar{B}$.

Laws of Set Theory:

Commutative laws:

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

Associative Laws :

$$A \cup (B \cup C) = (A \cup B) \cup C.$$
$$A \cap (B \cap C) = (A \cap B) \cap C.$$

Distributive Laws :

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Difference Laws :

$$A - B = A \cap \overline{B}$$
$$A - B = A - (A \cap B)$$
$$= (A \cup B) - B$$

Complementary Laws :

$$A \cup A^c = S$$
$$A \cup S = S$$
$$A \cup \phi = A$$

De-Morgan's Laws of complementation

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

Events as Sets :

Sample space : The set of all possible outcomes of a random experiment is known as the sample space and is denoted by S.

Events : Off all the possible outcomes in the sample space of a random experiment, some outcomes satisfy a specified description, which we call as event.

$A \cup B$: An event which represents the happening of at least one of the events A and B.

$A \cap B$: An event which represents the simultaneous happening of ~~the~~ both the events A and B.

$\bar{A}$: A does not happen.

$\bar{A} \cap \bar{B}$: Neither A nor B happens.

$\bar{A} \cap B$: A does not happen but B happens.

$(A \cap \bar{B}) \cup (\bar{A} \cap B)$: Exactly one of the two events A and B happens.

— The probability of occurance of at least one of the two events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

— If the events A and B are mutually disjoint i.e $A \cap B = \phi$

$$P(A \cap B) = \frac{n(A \cap B)}{N} = \frac{n(\phi)}{N} = 0 .$$

— The Probability of happening of any one of the two mutually disjoint events is equal to the sum of their individual probabilities.

$$P(A \cup B) = P(A) + P(B).$$

— The probability of simultaneous happening of two events A and B.

$$P(A \cap B) = P(A) \cdot P(B/A) ; \; P(A) \neq 0$$

$$P(B \cap A) = P(B) \cdot P(A/B) ; \; P(B) \neq 0$$

$P(B/A)$ is the conditional probability of happening of B under the condition that A has happened and $P(A/B)$ is the conditional probability of happening A under the condition that B has happened.

Independent Events:

$$P(A/B) = P(A) \text{ and } P(B/A) = P(B).$$

Multiplication theorem of independent events:

$$P(A \cap B) = P(A) \cdot P(B).$$

- $P(\bar{A}) = 1 - P(A)$
- $P(\bar{A} \cap B) = P(B) - P(A \cap B)$
- $P(A \cap \bar{B}) = P(A) - P(A \cap B)$
- If $A \subset B \Rightarrow P(A) \leq P(B)$.

Reference:

Fundamentals of Statistics by S.C. Gupta.

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